Event-Based Methods for the Control of Linear Time-Invariant Systems

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" This work has been partially supported by the LabEx PERSYVAL-Lab (ANR-11-LABX-0025-01) funded by the French program Investissements d'avenir "

Outline

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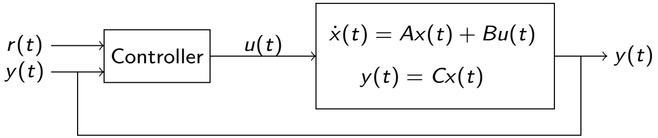
Definition of Event-Based Control

Classical Control



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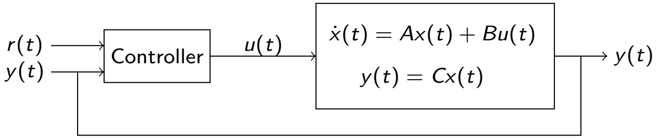
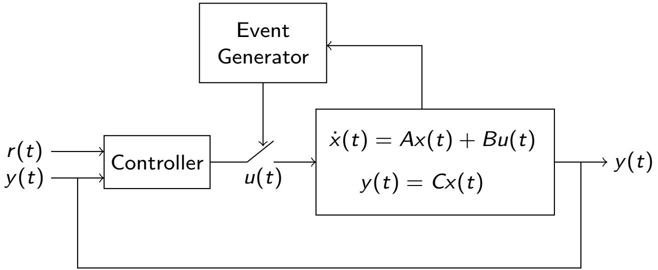


Event-Based Control



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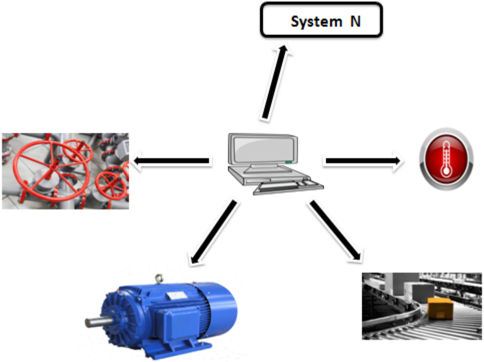
Objectives of the Event-Based Approach



- Relieving the load on the communication channels,
- Reducing the computational load on the CPU,
- Reducing energy consumption.

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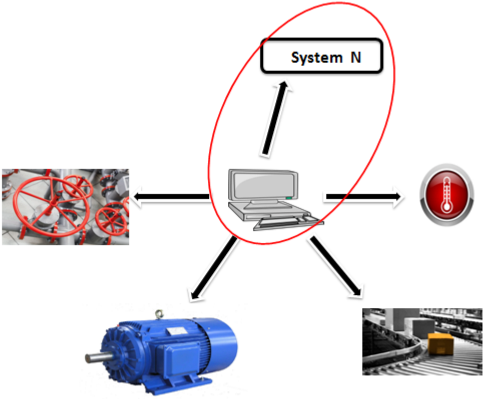

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Objective of our Event-Based Algorithms

Many methods have been proposed in the literature

- Tabuada, 2007,
- Lunze, 2009,
- Meslem & Prieur 2015.

Our objectives

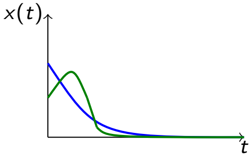
- Provide event-based solutions for the tracking problem
- Simplify the previous approaches and reduce further the controller-system interactions.

Outline

Problem Statement

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t)$$

$\longrightarrow$



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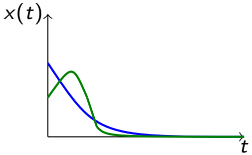

At $t = t_k$, an event occurs

$$u(t_k) = -Kx(t_k).$$

such that $(A - BK)$ is Hurwitz.
When $t \in]t_k, t_{k+1}[$,

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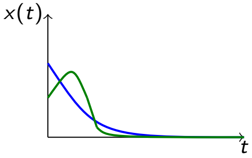

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When $t \in]t_k, t_{k+1}[$,

$$u(t) = u(t_k).$$

We define a Lyapunov-like function

$$V(x(t)) = x^T(t)Px(t),$$

where P is solution to $(A - BK)^T P + P(A - BK) = Q$.

Event-triggering Conditions

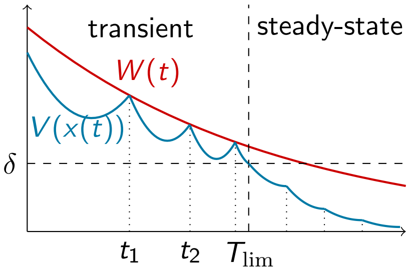
Transient Time Interval

$t < T_{\lim}$ and $T_{\lim} = min\{t | V(x(t)) = \delta\}$.
We define a threshold function

$$W(t) = V(x_0)e^{-\alpha t},$$

where $x_0 = x(t_0 = 0)$
and $\alpha \in]0, \lambda_{\max}(Q, P)[$.

$$t_{k+1} = \inf\{t > t_k, \quad V(x(t)) = W(t)\}.$$



Steady-state Regime

$t \geq T_{\lim}$

$$t_{k+1} = \inf\{t > t_k, \quad \frac{dV(x)}{dt} = 0\}.$$

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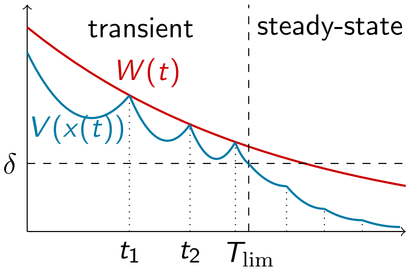

We can prove that:

- The system is asymptotically stable,
- $\exists$ a min inter-sample time.

Numerical Results

Consider the second order LTI system

$$\frac{dx}{dt} = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t),$$

where

$$u(t) = \begin{bmatrix} -1.7329 & -5.6667 \end{bmatrix} x(t).$$

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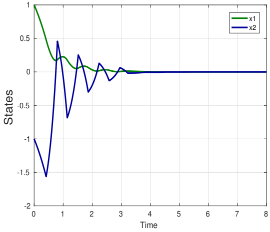

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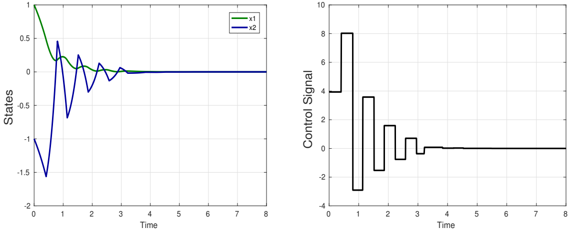

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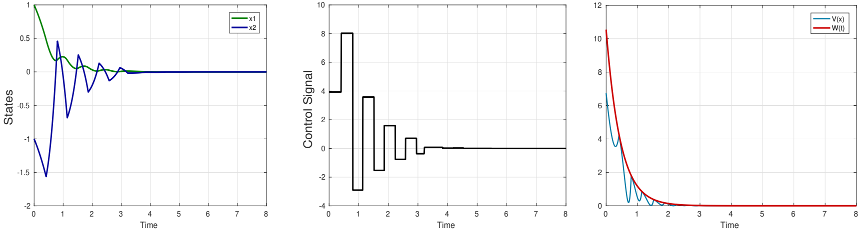

16 updates in 8.10^4 sampling instants.

Outline

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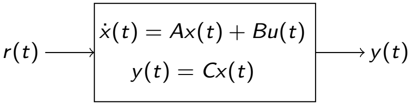

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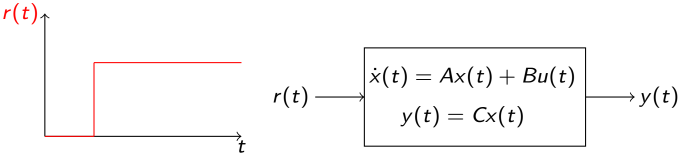

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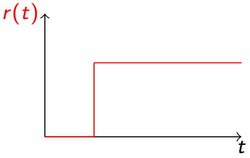
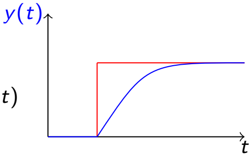


$r(t)$ →

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t)$$

→ $y(t)$



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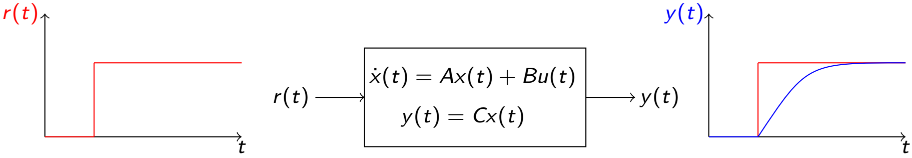

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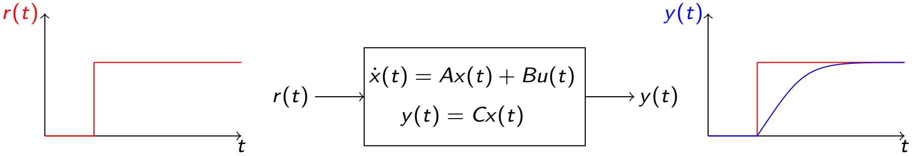

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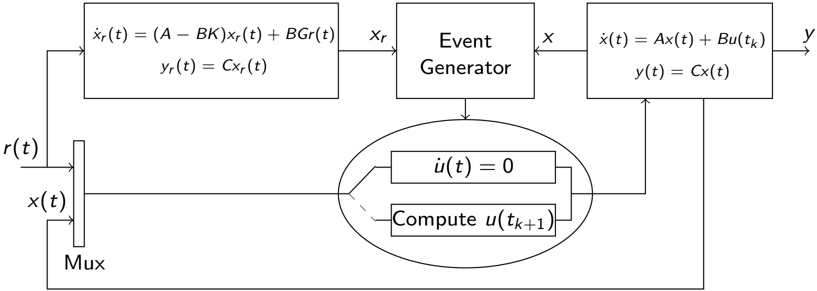
Event-triggering Conditions

Classical version of our LTI system is taken as reference.



$$e(t) = x(t) - x_r(t) \rightarrow 0$$

Event-triggering Conditions : Cont'd

$$V(e(t)) = e(t)^T \ P \ e(t),$$

where $P > 0$ and satisfies the Lyapunov equation

$$(A - BK)^T P + P(A - BK) = Q, \qquad Q < 0.$$

Event-triggering Conditions : Cont'd

$$V(e(t)) = e(t)^T \; P \; e(t),$$

where $P > 0$ and satisfies the Lyapunov equation

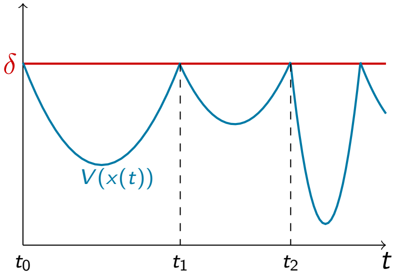
$$(A - BK)^T P + P(A - BK) = Q, \qquad Q < 0.$$

$$W(t) = \delta.$$

$$t_{k+1} = \inf\{t > t_k, \;\; V(x(t)) = W(t)\}.$$

We can prove

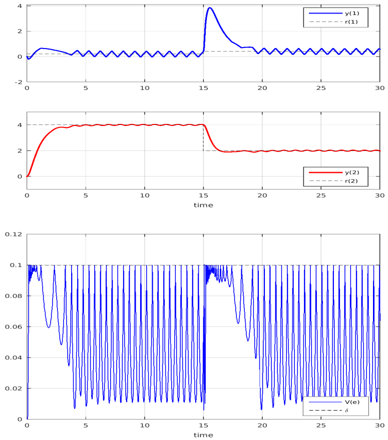
- Practical stability,
- Existence of inter-sample time.



Numerical Results

A simplified model for an aircraft while under cruise control with

- Four states,
- Two inputs: rudder and aileron deflections,
- Two outputs : the bank angle and the yaw rate.



75 updates in 3.10^4 sampling instants.

Outline

Perspective

Work in Progress

- What happens in the presence of a disturbance.
- Extension to the nonlinear case.

Future Work

- A self-triggered approach.

Publications

Thesis started : 01/10/2015.

Publications

- “ Event-based sampling algorithm for setpoint tracking unsing a state-feedback controller,” in Second International Conference on Event-Based Control, Communications, and Signal Processing. Krakow, Poland: IEEE, June 2016.
- “ Event-Triggered Stabilizing Controllers Based on an Exponentially Decreasing Threshold,” in Third International Conference on Event-Based Control, Communications, and Signal Processing. Funchal, Portugal: IEEE, May 2017.

Thank you for your attention!