# Target Identification using Electroreception

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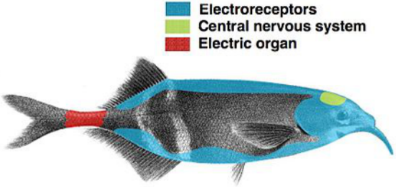
# Electroreception for weakly electric fish

Electroreception is the ability that special fish species use to recognize their environment.

- Electric organ: generate a stable, high-frequency, weak electric field.
- Electroreceptors: measure the transdermal potential modulations caused by a nearby target.
- Nervous system: perceive target's shape and location.




The electric fish, and its electric organs.

# Toward fish-inspired aquatic robots

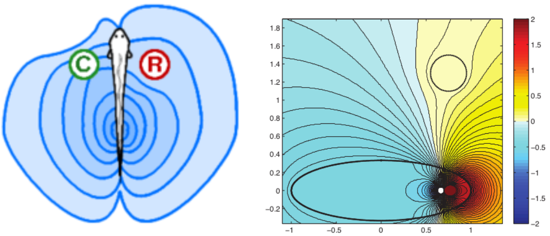
The objectives of the Phd Thesis are

i to derive a mathematical model for the Electroreception.

ii to study the inverse problem related to the Electroreception:

- to analyse how the identification procedure is sensitive to model and measurements errors (stability estimates).
- to solve numerically the inverse problem (convergence estimates).

iii to derive fast and efficient algorithms for real time electro-localization electric fish like aquatic robots.




The generated electric field by the fish and its interaction with a target.

# The weakly electric fish

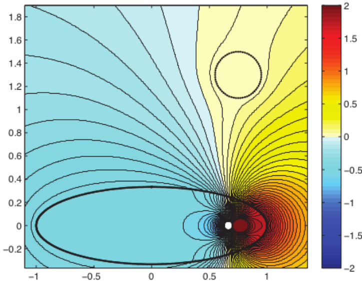
## The mathematical model:

Let

- $\Omega \subset \mathbb{R}^d$ be the electric fish.
- $\triangledown$ $k_0 > 0$ be the conductivity of the background $\mathbb{R}^d \setminus \overline{\Omega}$.
- $D \subset \mathbb{R}^d \setminus \overline{\Omega}$ be the target with conductivity $k(\omega) : [\underline{\omega}, \overline{\omega}] \to \mathbb{C} \setminus \mathbb{R}_-$, such that

$$\Sigma := \{k(\omega); \omega \in [\underline{\omega}, \overline{\omega}]\} ,$$

has an accumulation point in $\mathbb{C}$.
For example the empirical Drude model:




$$k(\omega) := \kappa_1 - \frac{\kappa_2}{\omega^2 + i\omega\kappa_3}, \tag{1}$$

where $\kappa_p > 0$ are constants that depend on the biological tissue of the target $D$.

# The weakly electric fish

The mathematical model:

Let

- $u(\cdot, \omega)$ be the electric potential field generated by the fish:

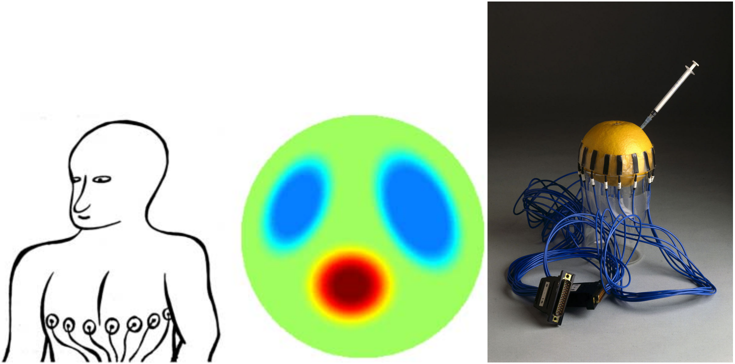
$$\begin{cases} \triangle u(x,\omega) = f(x) & \text{in } \Omega, \\ \partial_{\nu\Omega} u(x,\omega)|_- = 0 & \text{on } \partial\Omega, \\ \nabla \cdot [k_0 + (k(\omega) - k_0)\chi_D(x)]\nabla u(x,\omega) = 0 & \text{in } \mathbb{R}^d \setminus \overline{\Omega}, \\ u(x,\omega)|_+ - u(x,\omega)|_- = \xi \partial_{\nu\Omega} u(x,\omega)|_+ & \text{on } \partial\Omega, \\ |u(x,\omega)| = O(|x|^{1-d}) & \text{as } |x| \to \infty. \end{cases} \quad (2)$$

where

- wave-type electric signal: $\mathbf{f}(x,t) = f(x)\sum_n a_n e^{in\omega_0 t}$.
- $\omega = n\omega_0$ are the probing frequencies.
- $\chi_D$ is the characteristic function of the target $D$.
- $\xi$ the effective thickness of the fish skin $\partial\Omega$.

# Multifrequency Electrical Impedance Tomography

Similarities with the Multifrequency Electrical Impedance Tomography.



# The Multifrequency Electrical Impedance Tomography

The mathematical model:

Let

- $\Omega$ be the biological tissu under probe.
- $D \subset \Omega$ be the cancerous tissu with conductivity $k(\omega)$.
- $\Omega \setminus \overline{D}$ be the normal tissu with conductivity $k_0$.

- The electric potential $u(\cdot, \omega)$ solution

$$\begin{cases} \nabla \cdot (k_0 + (k(\omega) - k_0)\chi_D(x)) \nabla u(x,\omega) = 0 & \text{in} \quad \Omega, \\ k_0 \partial_{\nu_\Omega} u(x,\omega)(x) = f(x) & \text{on} \quad \partial\Omega, \\ \int_{\partial\Omega} u(x,\omega) ds = 0, & \end{cases} \tag{3}$$

  is generated by the input current $f(x)$ on $\partial\Omega$.

- The mfEIT inverse problem is to recover $D$ from measurements of $u(x,\omega)$ on $\partial\Omega$ for $\omega \in [\underline{\omega}, \overline{\omega}]$, $0 \leq \underline{\omega} < \overline{\omega}$, that is

$$u(x,\omega)|_{\partial\Omega}, \ \omega \in [\underline{\omega}, \overline{\omega}] \longrightarrow D$$

# Stability estimates for disks with a single frequency

**Theorem (Bonnetier-Triki-Tsou 2016)**

*Let $D$ and $\tilde{D}$ be two disks in $\Omega$. Denoting by $u$ (resp. $\tilde{u}$) the solution of (3) with the inclusion $D$ (resp. $\tilde{D}$). Let*

$$\varepsilon = \sup_{x \in \partial\Omega} |u - \tilde{u}|.$$

*Then, there exist constants $C > 0$ and $0 < \mu < 1$ such that,*

$$|D \triangle \tilde{D}| \leq C\varepsilon^{\mu}. \tag{4}$$

*Here, $\triangle$ denotes the symmetric difference and the constants $C, \mu$ depend only on $\Omega$ and $f$.*

# Spectral decomposition with multifrequency measurements

## Theorem (Ammari-Triki, 2016)

*Let $u(x,\omega)$ be the unique solution to the system (3). Then the following decomposition holds:*

$$u(x,\omega) = k_0^{-1}u_0(x) + \sum_{n=1}^{\infty} \frac{\int_{\partial\Omega} f(z)w_n^{\pm}(z)ds(z)}{k_0 + \lambda_n^{\pm}(k(\omega) - k_0)} w_n^{\pm}(x),$$
$$= k_0^{-1}u_0(x) + u_f(k(\omega), x), \tag{5}$$

*where $u_0(x) \in H^1_{\diamond}(\Omega)$ depends only on $f$ and $D$, and is the unique solution to*

$$\begin{cases} \triangle u = 0 & \text{in } \Omega \setminus \overline{D}, \\ \nabla u = 0 & \text{in } D, \\ \frac{\partial u}{\partial \nu} = f & \text{on } \partial\Omega, \\ \int_{\partial\Omega} uds = 0 \end{cases} \tag{6}$$

# Stability estimate with multifrequency measurements

**Theorem (Ammari-Triki, 2017)**

*Let $D$ and $\tilde{D}$ be two inclusions in $\Omega$. Denoting by $u$ (resp. $\tilde{u}$) the solution of (3) with the inclusion $D$ (resp. $\tilde{D}$). Let*

$$\varepsilon = \sup_{x \in \partial\Omega, \omega \in [\underline{\omega}, \overline{\omega}]} |u - \tilde{u}|.$$

*Then, there exist constants $C > 0$ and $0 < \tau < 1$ such that,*

$$|D \triangle \tilde{D}| \leq C\left(\frac{1}{\ln(\varepsilon^{-1})}\right)^{\tau}. \tag{7}$$

*Here, $\triangle$ denotes the symmetric difference and the constants $C, \tau$ depend only on $\Omega$ and $g$.*

# Optimization algorithm

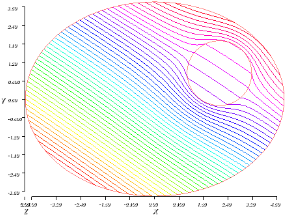
Our scheme is essentially based on the minimizing of the functional

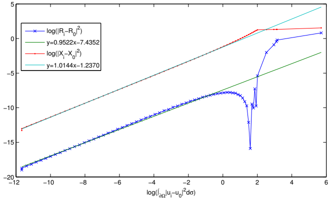
$$J(u) = \frac{1}{2}\int_{\partial\Omega} |u - u_{meas}|^2 ds(x),$$

where $u$ is the simulated solution of (3) and $u_{meas}$ is the measurement.

- FreeFem++ for numerical experiments.
- P2 finite elements for the numerical resolution of the PDEs.
- Spectral Fourier cut-off regularization method for the retrieval of the shape of $D$.

# Reconstruction results with a single frequency




We found in this case that $\mu = 0.96$.

# Reconstruction results with multifrequency measurements

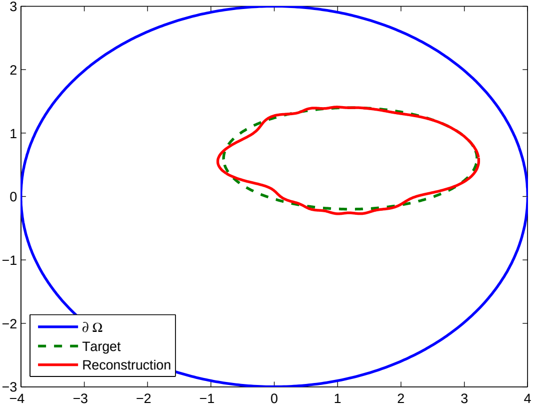



Figure: Reconstruction: ellipse

# Reconstruction results with multifrequency measurements

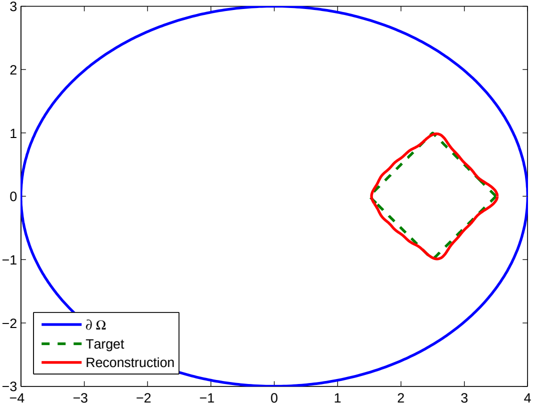



Figure: Reconstruction: square

# Reconstruction results with multifrequency measurements

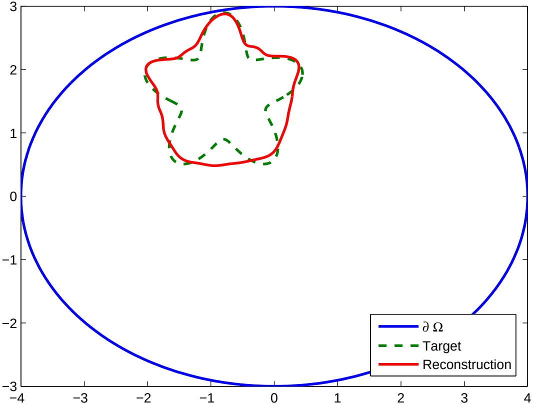



Figure: Reconstruction: concave domain

# Reconstruction results with multifrequency measurements

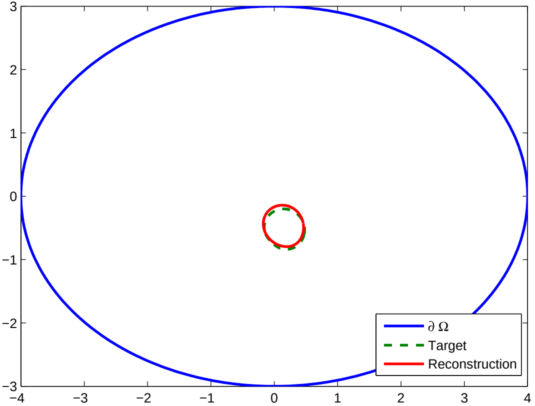
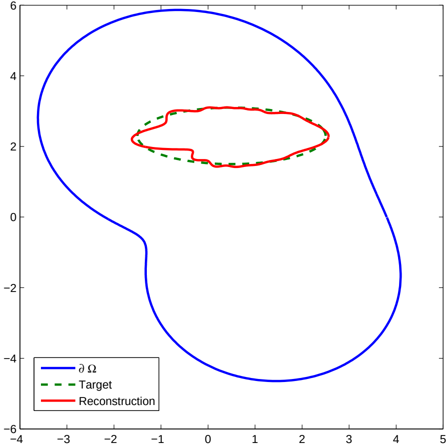



Figure: Reconstruction: small domain

# Reconstruction results with multifrequency measurements



# Conductivity profile reconstruction & Relative symmetric differences

Conductivity profile reconstruction

| • | real value | ellipse | square | concave | small | in $\Omega_2$ |
|---|---|---|---|---|---|---|
| $\kappa_1$ | 3 | 2.80971 | 3.36482 | 3.00287 | 6.65418 | 2.89787 |
| $\kappa_2$ | 2 | 1.79063 | 2.34197 | 1.96926 | 5.14671 | 1.86579 |
| $\kappa_3$ | 1 | 1.00212 | 0.987247 | 0.999658 | 1.13223 | 1.00446 |

Relative symmetric difference$= \dfrac{|D_{reconst} \triangle D_{target}|}{|D_{target}|}$.

| • | ellipse | square | concave | small | in $\Omega_2$ |
|---|---|---|---|---|---|
| values | 0.07055 | 0.12187 | 0.24299 | 0.19471 | 0.1205968 |

# References

- H. Ammari, F. Triki and C-H. Tsou, Numerical determination of anomalies in multifrequency electrical impedance tomography, submitted to SIAM Applied Mathematics (2017).
- H. Ammari and F. Triki. Identification of an inclusion in Multifrequency Electric Impedance Tomography, Communications in Partial Differential Equations, Issue 1, Volume 42, Pages 159-177, (2017).
- E. Bonnetier, F. Triki and C.H. Tsou. Eigenvalues of the Neumann-Poincaré operator for two inclusions with contact of order m: A numerical study, to appear in Journal of Computational Mathematics (2017).
- E. Bonnetier, F. Triki and C.H. Tsou. Inverse Condutivity Problem: A stable method to determine discs, *preprint 2017*

# Conclusion and perspectives

We have established stability estimates for the multifrequency impedance tomography. The numerical calculations seem to be in agreement with the theoretical results.

We plan

- to study the case with multiple inclusions $D_j$, $j \in J$.
- to extend the analysis to unbounded domains: the weak electric fish.
- to derive fast algorithms for identifying the inclusions within a set of given shapes.

*THANK YOU FOR YOUR ATTENTION!*