Positivity certificates for polynomials using amœbas

Supervision: Victor Magron[a] Xavier Allamigeon[b] Stéphane Gaubert[b]
Contacts: victor.magron@imag.fr xavier.allamigeon@inria.fr stephane.gaubert@inria.fr
Homepage: http://www-verimag.imag.fr/~magron/
Working place: Laboratoire VERIMAG, Grenoble

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Context Finding positivity certificates for a multivariate polynomial is a central problem in optimization. The method providing more accurate certificates consists of finding a sum of squares decomposition of this polynomial, which boils down to solving a convex optimization problem (called a *semidefinite* program). Even though such certificates can be computed in polynomial time (up to an additive error), this framework only provides sufficient conditions: a bivariate nonnegative polynomial is not necessarily a polynomial sum of squares.

Iliman et al. [2] recently introduced a new type of positivity certificates, relying on Archimedean amœbas. The latter mathematical objects have been studied for fifteen years and are the images of polynomial system solutions under the log-module map. Amœbas have various combinatorial properties related to polytopes. The framework proposed in [2] combines these properties with convex optimization methods to compute certificates in a more efficient way. However, this approach only applies to a restricted class of polynomials, whose supports are simplex Newton polytopes.

Objectives We believe that this approach could be extended to more general polynomial optimization problems, leading to the following two-fold research investigation:

1. identifying polynomial systems for which the framework from [2] can be applied
2. implementing a tool to perform numerical experiments relying on geometric programming (e.g. with the `cvx` [1] Matlab toolbox)

Required Skills

- Convex optimisation, geometric and semidefinite programming
- Linear algebra basics
- Programming with OCaml/C/C++/Matlab according to preference of the candidate

Working Context The Master student will be hosted by the Verimag laboratory, near Grenoble in the French Alps. The Grenoble area, in addition to the surrounding skiable mountains, features one of Europe's largest concentrations of academic/industrial research and development with a lot of students and a relatively-cosmopolite atmosphere. You can easily reach Lyon (1 hour), Geneva (1.5 hours), Torino (2 hours), Paris (3 hours by train) and Barcelona (6 hours).

A related PhD topic can be foreseen.

References

[1] M. Grant and S. Boyd. CVX: Matlab software for disciplined convex programming, version 2.1. http://cvxr.com/cvx, Mar. 2014.

[2] S. Iliman and T. de Wolff. Lower Bounds for Polynomials with Simplex Newton Polytopes Based on Geometric Programming. *ArXiv e-prints*, Feb. 2014.

[a]CNRS-VERIMAG Tempo team, Centre Equation 3, Office 15, 2 avenue de Vignate, 38610 Gières FRANCE
[b]INRIA Maxplus team, Centre de Mathématiques Appliquées de École Polytechnique, 91128 Palaiseau France