

# Analysis and Control of Partial Differential Equations

**Context** This work is in the line of research with the following issue: *how to develop new convex optimization techniques based on semidefinite programming (SDP) and real algebraic geometry to solve optimal control problems (OCP) in a nonlinear setting.*

Recently, several research efforts allowed to solve numerically certain optimal control problems with polynomial data. The general idea is to reformulate such a nonlinear problem into an infinite dimensional linear optimization problem, called *primal* over the moments of so-called *occupation measures*. This is inspired from a line of research initiated by Rubio [9]. This problem being convex but infinite dimensional, one way to handle it in practice is to solve a hierarchy of convergent semidefinite *relaxations*, each relaxation being solved with SDP numerical solvers. This approach allows to obtain sub-optimal polynomial solutions of the value function of the OCP, this function being a viscosity solution of a nonlinear Hamilton-Jacobi-Bellman partial differential equation (PDE). Each sub-optimal solution is provided through solving a *strengthening* of the dual sums-of-squares problem, associated to the primal reformulation over moments of occupation measures.

Further works allowed to apply the same framework to numerous problems related to optimal control:

- impulsive linear or nonlinear systems [2] or switch systems [3]
- certified over approximations of region of attraction [5] for polynomial systems or image set of polynomial dynamics [7]
- synthesis of control laws for infinite horizon problems [6]

**Goals** The aim of this internship is to extend the SDP framework to handle a more general class of control problems involving PDEs, including wave equations. Some promising research direction would be to relate results coming from differential geometry and flatness of nonlinear systems could be applied to the SDP approach in order to obtain certified approximations of control laws for such nonlinear polynomial PDE, when state and control fulfill semialgebraic constraints. Another track of interest includes the use of two operators allowing to linearize nonlinear (polynomial) systems, namely the Koopman and Perron-Frobenius operators [1]. We intend to rely on measures supported over infinite dimensional vector spaces [8] (by contrast with previous work where the support is usually a finite dimensional compact semialgebraic set). One application is the study of OCP involving wave propagation equations.

The numerical results will be obtained through implementation of a practical software package to compute converging approximations for these OCP problems. Another practical goal will be to establish comparisons with existent methods relying on Lyapunov polynomial functions, obtained through solving sums of squares problems [4].

**Working Context** The internship will be co-advised by Victor Magron (CNRS Verimag) and Christophe Prieur (CNRS Gipsa-Lab). The Master student will be hosted by the Verimag laboratory, near Grenoble in the French Alps. The Grenoble area, in addition to the surrounding skiable mountains, features one of Europe's largest concentrations of academic/industrial research and development with a lot of students and a relatively-cosmopolite atmosphere. You can easily reach Lyon (1 hour), Geneva (1.5 hours), Torino (2 hours), Paris (3 hours by train) and Barcelona (6 hours).

**Required Skills** Motivated candidates should hold a Bachelor degree and have a solid background in **either** optimization, control, electrical engineering. Good programming skills are also required. The candidates are kindly asked to send an e-mail with "Master candidate" in the title, a CV and motivation letter to [victor.magron@imag.fr](mailto:victor.magron@imag.fr) and [christophe.prieur@gipsa-lab.fr](mailto:christophe.prieur@gipsa-lab.fr). Knowledge of French is advantageous but does not constitute a pre-requisite.

**A related PhD topic can be foreseen.**

## References

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